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THE COLLAPSE OF SHORT THIN TUBES

BY

A. P. CARMAN

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PROFESSOR OF PHYSICS

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I. Results of Previous Experiments on Collapse of Tubes

1. *Fairbairn's Formula.*—The problem involved in the study of the collapse of tubes is to find an equation by the application of which the collapsing pressure of a given tube can be calculated when the dimensions of the tube and the elastic properties of the material are known. The first person to attempt a solution of the problem was the noted British engineer, Sir William Fairbairn. In 1858, Fairbairn published in the Philosophical Transactions of the Royal Society of London a paper describing experiments on the collapse of about twenty-five tubes. Fairbairn's work was done at the suggestion and with the aid of the Royal Society, and the British Association for the Advancement of Science. The immediate cause was the need of such knowledge in the design of steam boilers. The tubes were "composed of a single thin iron plate bent to the required form upon a mandril and riveted and also brazed to prevent leakage into the interior." The ends were closed by means of iron disks or plugs, and the tube was placed in a large iron cylinder where it was subjected to hydraulic pressure. The interior of the tested tube was connected with the atmosphere at the upper end. The pressure was produced by a hydraulic pump and was measured by ordinary steam gauges. The diameters of the tubes collapsed were 4, 6, 8, 9, 10, and 12 inches; the lengths ranged from 19 to 60 inches; and the wall thickness was .043 of an inch for each tube. The tubes were what we describe to-day as "short" and "thin."

Fairbairn summed up the results of his experiments in the well-known formula:

$$P = 9{,}675{,}600 \frac{t^{2.19}}{l\,d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

"which is," he says, "the general formula for calculating the strength of wrought iron tubes subjected to external pressure within the limits indicated by the experiments, that is, provided their length is not less than 1.5 feet and not greater than 10 feet." In this formula, as in other formulas presented in this bulletin, P is collapsing pressure in pounds per square inch; t is thickness of wall in inches; l is length of tube in inches; d is diameter in inches. It is noteworthy in the history of the subject that Fairbairn's formula in some form was the only available formula for engineers for nearly fifty years, and that his experiments seem to have been the only experiments made in this field prior to 1905 and 1906. Meanwhile, the materials and methods of construction, and the uses of tubes had changed so that Fairbairn's constants were wholly out of date.

In addition, Fairbairn's tubes, as has since been shown, were below a certain "critical length"; and, the fact of a minimum critical length not being known, the formula was generally assumed to be applicable to tubes of all lengths. Various modifications were proposed for the formula* but practically all of them were based on Fairbairn's data.

2. *Experiments of Carman.*—The first experiments on the collapse of tubes after Fairbairn's were those of A. P. Carman, which were reported and discussed in a paper read before the American Physical Society in April, 1905, and published in the Physical Review, Vol. XXI. These experiments were conducted with small seamless brass tubes and showed "that there is a minimum length for each tube above which the collapsing pressure is constant. Again," still quoting this paper, "we see that for lengths less than this critical minimum length, the collapsing pressure rises rapidly. As definitely as can be determined from these small tubes, the collapsing pressure varies inversely as the length, for lengths less than the critical length." It was further pointed out that Fairbairn's tubes were all less than the critical length. "These experiments on small brass tubes have shown that formulas of the Fairbairn type are inadequate, and that for tubes of sufficient length, a formula of the type proposed by Bryan and Love is more nearly true." The formula referred to is that first given by Professor G. H. Bryan† of Cambridge, England, and later deduced by other methods by A. B. Bassett‡ and A. E. H. Love§. The formula is given for long thin tubes, and is

$$P = 2E \frac{m^2}{m^2 - 1} \frac{t^3}{d^3} \qquad (2)$$

where P is the pressure of collapse, t is the thickness of wall, d is the mean diameter, E is Young's modulus, and $\frac{l}{m}$ is Poisson's ratio.

*See Bibliography.

†Proceedings Cambridge Philosophical Society, Vol. VI.

‡Philosophical Magazine, September, 1892.

§Mathematical Theory of Elasticity, Vol. 2, p. 319, 1903.

3. *Results Obtained by Stewart, and by Carman and Carr.*— In 1906, two papers appeared on the collapse of tubes; one by Professor R. W. Stewart* read before the American Society of Mechanical Engineers, giving the results of a large number of experiments on lap-welded steel tubes; and the other by Carman and Carr† in a bulletin of the Engineering Experiment Station of the University of Illinois, describing experiments on seamless steel, lap-welded steel, and brass tubes. All of these experiments were made on tubes longer than the "critical length," little attention being paid to the law of lengths except to see that the tubes were longer than the critical minimum length.

The results obtained by Carman and Carr were stated as follows:

"The portion of a long tube affected by the collapse under hydraulic pressure is generally not greater than twelve times the diameter; for greater lengths the collapsing pressure is independent of the length. The law according to which the collapsing pressure varies inversely as the length, is true only for very short tubes; i. e., tubes shorter than a certain critical 'minimum length' which in most cases is from four to six times the diameter.

" For long thin tubes, i. e., for values of $\frac{t}{d}$ below about .025, the formula $P = k\left(\frac{t}{d}\right)^3$ is very nearly true." The constants for brass and seamless tubes were calculated from the experimental data, and the formulas stated as follows:

a. For thin brass tubes

$$P = 25{,}150{,}000 \left(\frac{t}{d}\right)^3 \qquad (3)$$

b. For thin seamless steel tubes

$$P = 50{,}200{,}000 \left(\frac{t}{d}\right)^3 \qquad (4)$$

" For moderately 'thick' tubes, that is, for tubes having values of $\frac{t}{d}$ between .03 and .07, an equation of the form

$$P = k\frac{t}{d} - c \qquad (5)$$

*Transactions of the American Society of Mechanical Engineers, 1906.

†" Resistance of Tubes to Collapse." Univ. of Ill. Eng. Exp. Sta. Bul. 5, 1906.

was found to satisfy the experimental data, the formula becoming:

a. For brass

$$P = 93{,}365 \frac{t}{d} - 2474 \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

b. For seamless cold drawn steel

$$P = 95{,}520 \frac{t}{d} - 2090 \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

c. For lap-welded steel

$$P = 83{,}290 \frac{t}{d} - 1025 \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)\text{''}$$

Professor Stewart found similar equations and constants from his experiments on lap-welded steel tubes.

Thus from theory and from experiments, fairly satisfactory equations for long thin tubes and for long tubes of moderate thickness have been reached. The terms ''long,'' ''thin,'' and moderately ''thick'' are used as previously defined. The equation for long, moderately thick tubes is empirical both in its constants and its form. The equation for long thin tubes is that of Bryan except that the constant is about 30 per cent less than the theoretical value, probably due, as suggested by R. V. Southwell,* to the material being beyond the elastic limits.

4. *Investigations of Southwell, and of Cook.*—Within the last three years, there has been considerable interest in the law of the collapse of ''short'' thin tubes, that is, in the form of the pressure-length curve near and inside the critical length. The practical interest in this part of the curve came first from the problem of spacing ''collapse rings'' in boiler flues. Another practical interest in the problem is found in the collapse of steel flumes by atmospheric pressure when the water is suddenly let out by accident and the internal pressure is reduced almost to zero.

The theoretical interest comes from the very important mathematical investigations of R. V. Southwell, Fellow of Trinity College, Cambridge, England, discussed in a paper entitled ''On the General Theory of Elastic Stability.'' This paper was read before

*Philosophical Magazine, p. 504, September, 1913.

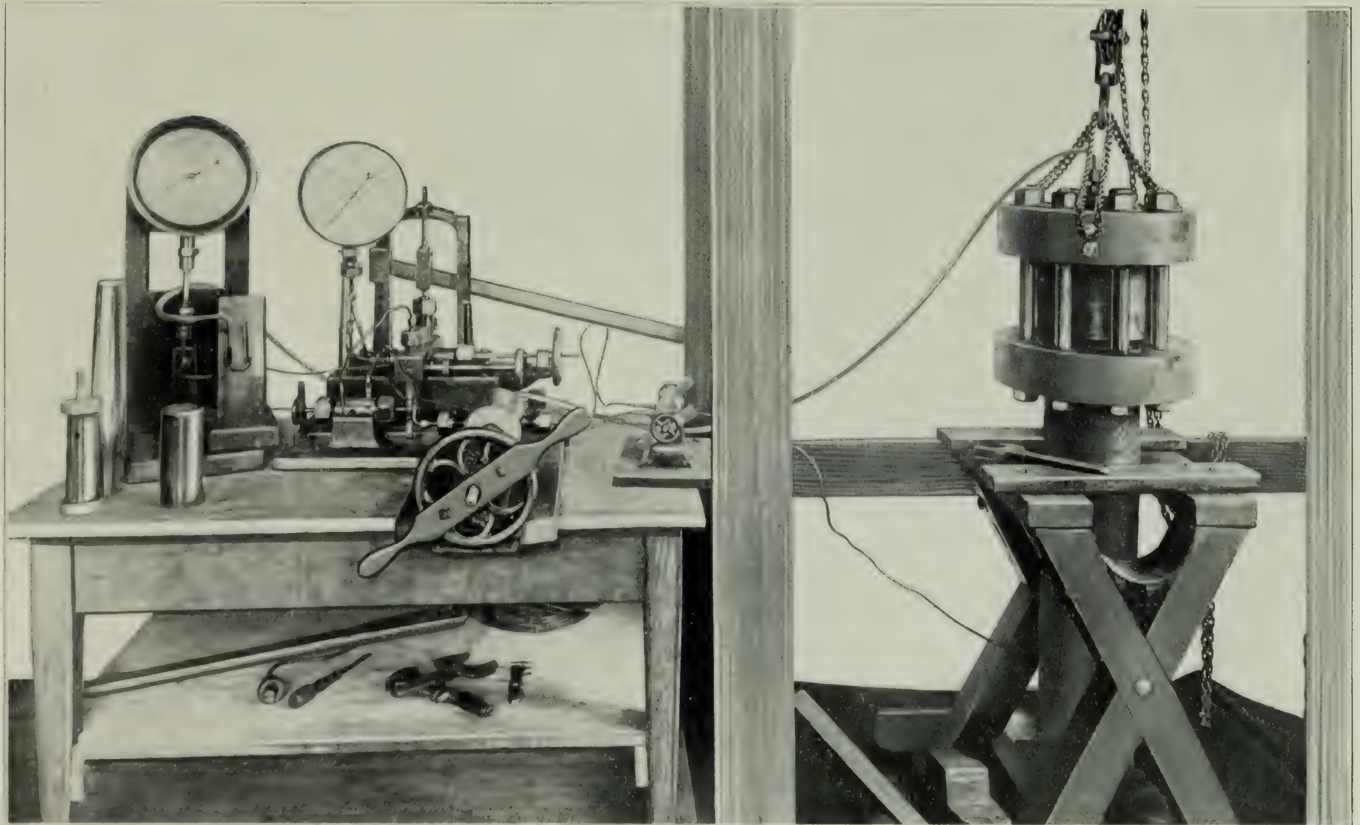

Fig. 1. Apparatus Arrangement. Pump and Gauges to Left, Nickel-Steel Cylinder to Right

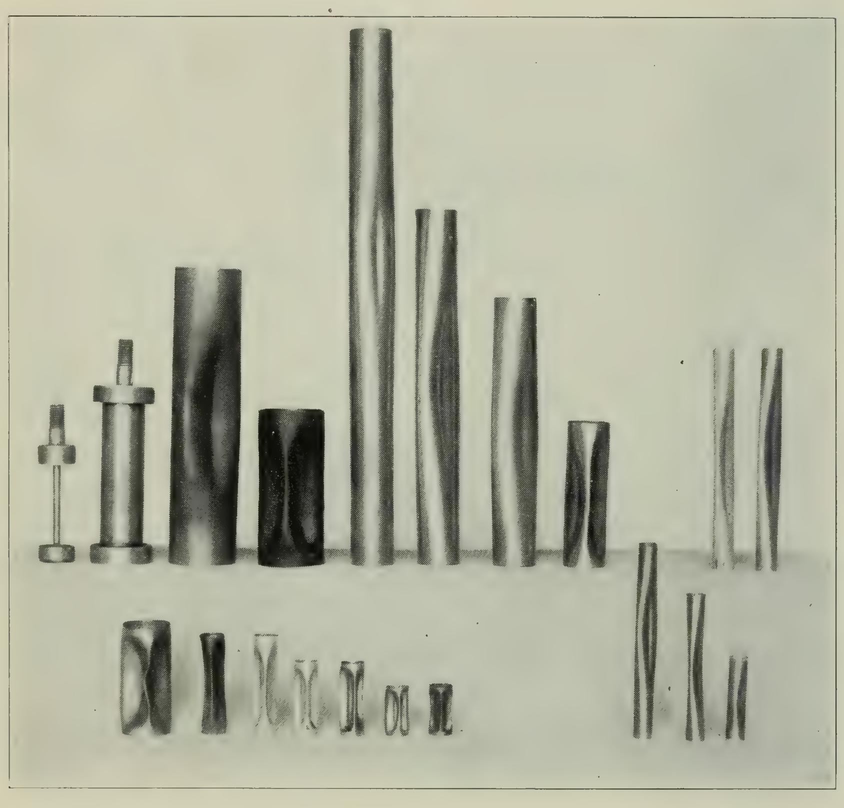

FIG. 2. TUBES THAT HAVE BEEN COLLAPSED. ON THE LEFT THE MOUNTING ARRANGEMENT IS SHOWN. LONG TUBES COLLAPSE IN TWO LOBES, SHORTER TUBES INTO THREE LOBES, AND STILL SHORTER TUBES INTO FOUR LOBES

the Royal Society of London in January, 1913.* In it Southwell developed the following formula for the collapse of thin tubes:

$$P = 2E\frac{t}{d}\left[\frac{Z}{k^4\,(k^2-1)}\,\frac{d^4}{l^4}+\frac{1}{3}\,\frac{m^2}{m^2-1}\,(k^2-1)\,\frac{t^2}{d^2}\right] \quad . \qquad (9)$$

where P is the collapsing pressure, E is Young's modulus, $\frac{l}{m}$ is Poisson's ratio, Z is a constant depending upon the end constraints, l, d,

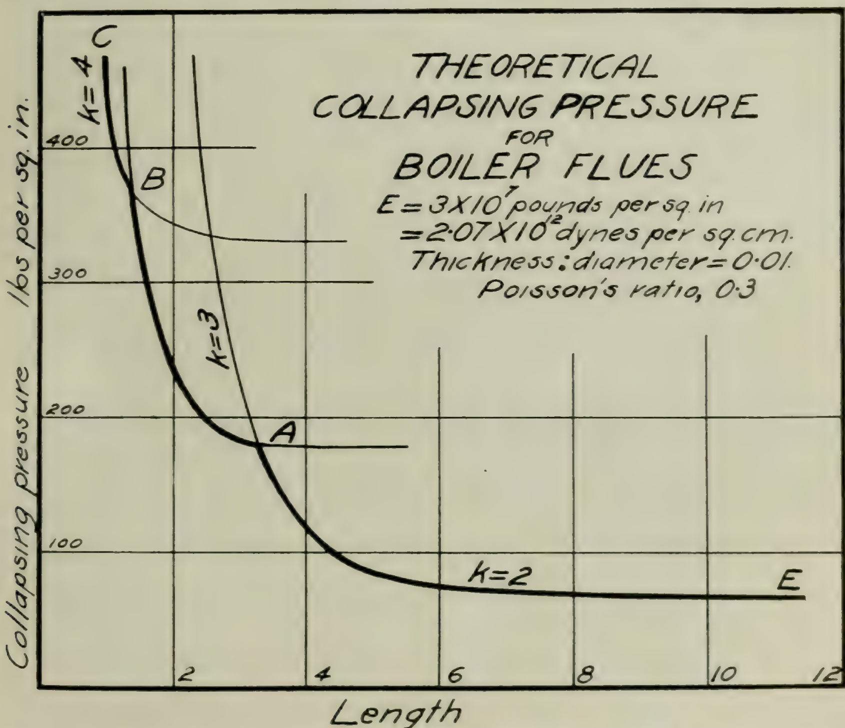

FIG. 3. CURVE OF SOUTHWELL'S EQUATION REPRODUCED FROM PHILOSOPHICAL TRANSACTIONS, 1913

and t are the length, diameter, and wall thickness, and k represents the number of lobes into which the tube collapses. It is known that long tubes collapse into two lobes, shorter tubes into three lobes, and still shorter into four lobes (See Fig. 2). This formula is represented by a family of curves, corresponding to the value of 2, 3, and 4, for k. Fig. 3 is reproduced from Southwell's paper. It shows the curves

*Philosophical Transactions of the Royal Society of London, Vol. 213A, pp. 187-244.

for two, three, and four lobes, giving the relation between collapsing pressure and length; values of Young's modulus and Poisson's ratio are assumed as noted on the curve. No simple value for Z is stated by Southwell. "From an inspection of the different curves, we see," writes Southwell, "that long tubes will always tend to collapse into the two-lobed form, since the curve for $k=2$ then gives the least value for the collapsing pressure, but that at a length corresponding to the point A, the three-lobed distortion becomes natural to the tube; and that for shorter lengths still, of which the point B gives the upper limit, the four-lobed form requires least pressure for its maintenance. Thus the true curve connecting pressure and length is the discontinuous curve $CBAE$, shown in the diagram by a thickened line." Then according to Southwell's theory, the curves which are ordinarily drawn to represent the results of experiments, such as the curves in Figs. 8 to 14, are really the envelopes of series of curves. Southwell made some experiments which seem to confirm his idea of the pressure-length curve. Our own experiments have not given any decisive proof of this feature of Southwell's formula. It is to be noted that for long tubes, that is, where $k=2$, Southwell's formula reduces to the Bryan formula.

Southwell has further discussed the theory of the collapse of tubes in three articles in the Philosophical Magazine for 1913 and 1915.* The third of these articles is largely a discussion of his theory with reference to some very interesting experiments on the collapse of short tubes by Gilbert Cook of the University of Manchester.†

Southwell's formula differs from the previous formula of Bryan in giving an expression for the collapsing pressures for lengths less than the "critical length." He deduces as an expression for the "critical length," $L = k\sqrt{\frac{d^3}{t}}$, "$k$ being some constant depending upon the type of the end constraints."‡

The Southwell formula further makes the envelope of the pressure-length curve a hyperbola for lengths less than the critical length. Hence, if the critical length can be determined, the curve can be drawn and the strength of a short tube calculated. The experiments

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*Philosophical Magazine, pp. 687-698, May, 1913; pp. 502-511, September, 1913; pp. 67-77, January, 1915.

†Philosophical Magazine, pp. 51-56, July, 1914.

‡Philosophical Transactions, Vol. 213A, p. 227.

of Carman and of Stewart indicated a length of six diameters as the critical length.

Cook's experiments were made to determine the strength of short tubes and, as a part of the investigation, to determine the ''critical length.'' He gives the results of the collapses of thirty-nine ''solid drawn'' steel tubes, all of three inches internal diameter, of values of $\frac{t}{d}$ varying between .0097 and .0206, and of lengths from 2.08 inches to 12.71 inches. Unfortunately Cook's apparatus limited him to tubes of less than thirteen inches in length, that is, to lengths of about four diameters, so that his curves do not reach the important bends or ''critical'' points. He, however, concludes from estimates that ''the critical length varies from thirteen to eighteen times the diameter,'' and that it is given by Southwell's formula $L = k\sqrt{\frac{d^3}{t}}$, with $k = 1.73$ for his steel tubes. Southwell in his discussion of Cook's work and the ''critical length'' says,* ''Both theory and experiment suggest that the length of a tube sensibly affects its resistance to external pressure only in the case of comparatively short tubes, and the earliest definitions of the term 'critical length,' given almost simultaneously by Professor A. E. H. Love as 'the least length for which collapse is possible under the critical pressure,' and by A. P. Carman as a 'minimum length, beyond which the resistance of a tube to collapse is independent of the length,' were in recognition of this fact. Professor Carman concluded further from the early experiments of Fairbairn and from others which he himself conducted, that 'the collapsing pressure varies inversely as the length, for lengths less than the critical length.' That is to say, the curve suggested by him as expressing the experimental relation between collapsing pressure and length for a tube of given thickness and diameter consists of two discontinuous branches; a straight line, representing constant collapsing pressure for all lengths above the critical length, and a rectangular hyperbola intersecting this line at a point corresponding to the critical length.

''If these views were adopted, the critical length for any definite size of tube may be determined from experiments by estimating: (1) the straight line, parallel to the axis of length, which best represents the collapsing pressure for tubes of considerable length, and (2) the

*Philosophical Magazine, p. 68, January, 1915.

hyperbola which agrees best with the results for the shorter tubes; their point of intersection gives the required value. This is substantially the procedure adopted by Mr. Cook, who finds that within the range of his experiments the critical length L, thus defined, is given satisfactorily by the formula.

$$L = 1.73\sqrt{\frac{d^3}{t}} \qquad (10)$$

"Two of the five curves upon which Cook bases his conclusions are

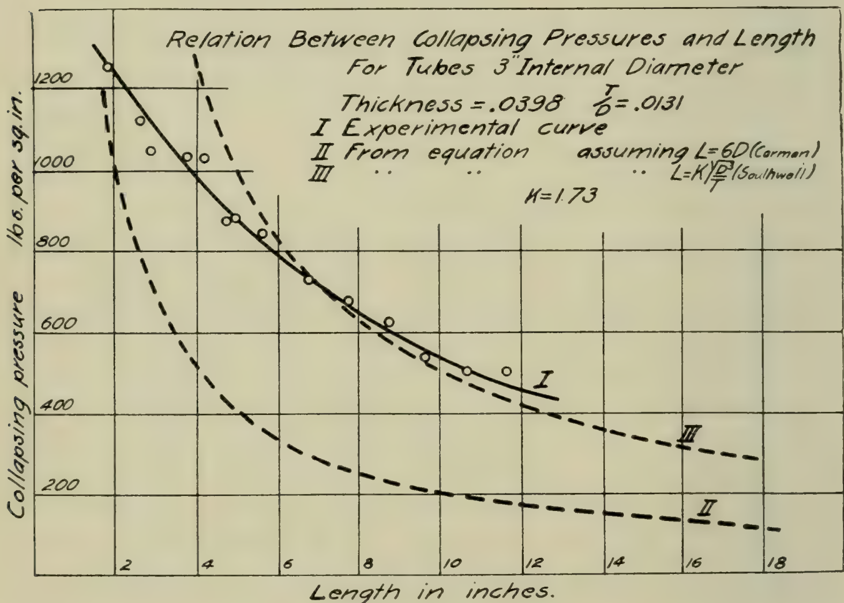

FIG. 4. PRESSURE-LENGTH CURVES SHOWING EXPERIMENTAL AND CALCULATED VALUES ACCORDING TO GILBERT COOK. REPRODUCED FROM PHILOSOPHICAL MAGAZINE, 1914

shown in Figs. 4 and 5 which are reproduced from his article. Agreement of the curve with this theory is not close, particularly for the thicker tubes. Although Cook suggests the given formula, he frankly says that 'the tests cannot be regarded as sufficient in number or covering enough range of dimensions to confirm definitely the equation $L = 1.73\sqrt{\frac{d^3}{t}}$ Though there is considerable amount of data for a law on long thin tubes and for long moderately thick tubes, there are few experimental results on the collapse of short tubes. The following experiments were undertaken in order to get more facts on the strength of short tubes of steel and of brass. To these have also been added some experiments on tubes of aluminum, glass, and hard rubber. The results obtained with the three last named materials not only have an interest in themselves, but also may have a theoretical interest in future discussions of formulas on account of the different elastic constants involved.

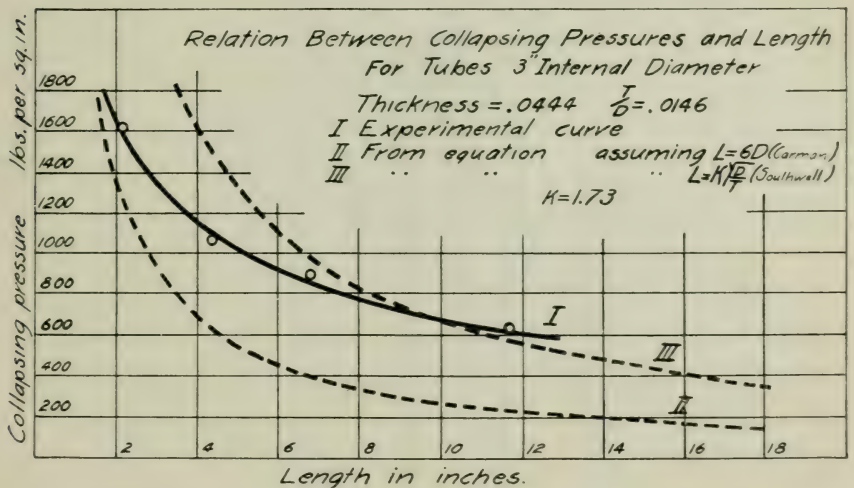

FIG. 5. PRESSURE-LENGTH CURVES SHOWING EXPERIMENTAL AND CALCULATED VALUES ACCORDING TO GILBERT COOK. REPRODUCED FROM PHILOSOPHICAL MAGAZINE, 1914

A large part of the experimental work described in this paper has been done by Stetfan F. Tanabe, M. S., Research Fellow of the Engineering Experiment Station. Credit is due him also for many of the details of the apparatus and methods.

II. METHODS AND RESULTS OF EXPERIMENTS ON THE COLLAPSE OF SHORT THIN TUBES

5. *Apparatus.*—The apparatus was for the most part that which the author used a number of years ago in previous experiments of this kind. The hydraulic pressure was produced in a stout nickel-steel tube or receptacle 40 inches in length, 5 inches in internal diam-

eter, and 7 inches in external diameter. The lower end was closed by a cast-iron plug, and the upper end by a heavy steel disk or cover which

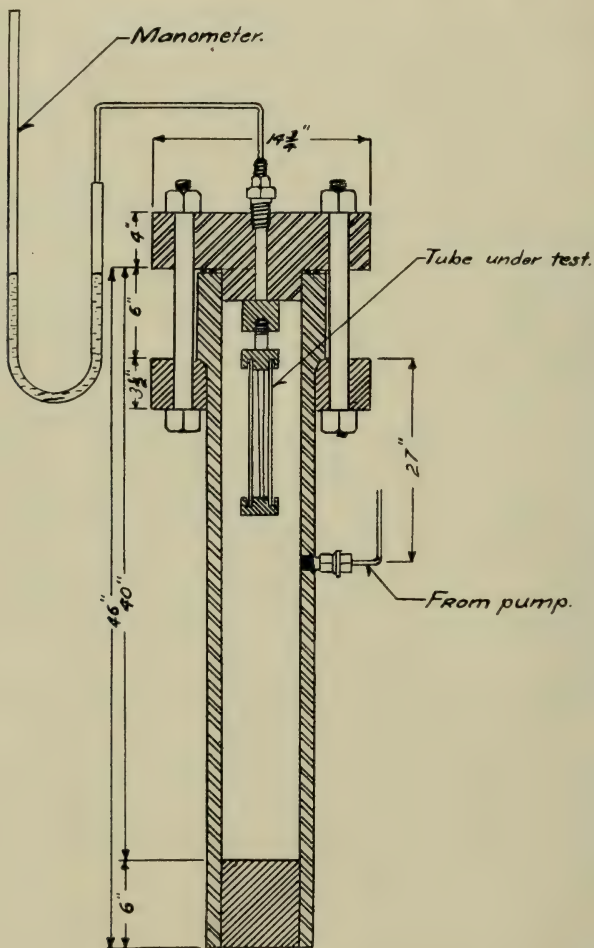

Fig. 6. Section of the Nickel-Steel Tube in which the Tubes were Collapsed by Hydraulic Pressure

was held in place by eight circular bolts as shown in Figs. 5 and 6. A lead gasket with circular grooves in the end face of the tube prevented leakage even at the highest pressures. This tube was held in a vertical position, so that the heavy steel cover could be conveniently handled by a chain hoist. The hydraulic pressure was produced by a Cailletet pump, made by the Société Génevoise, and by a screw plunger, which was made in the department shop. The pressure was raised nearly to the required amount by the pump, and then brought

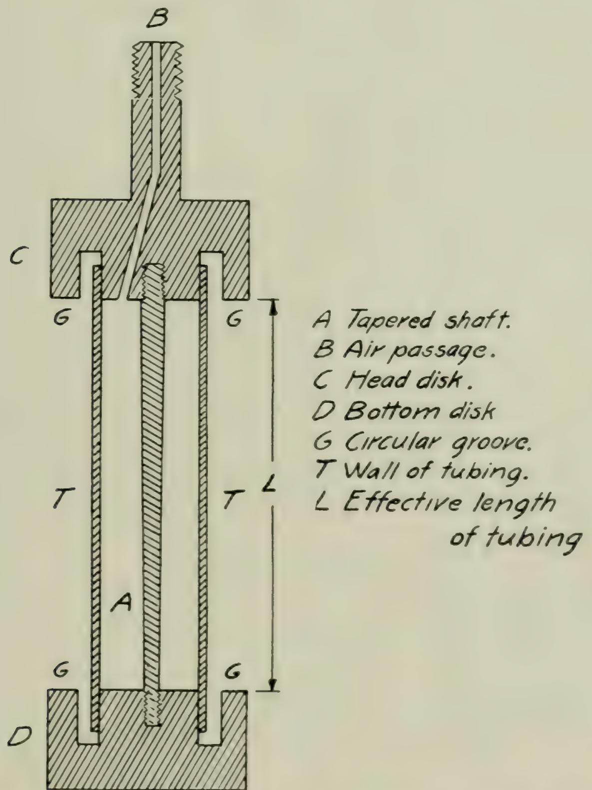

FIG. 7. CROSS-SECTION OF SPINDLE FOR MOUNTING A TUBE FOR COLLAPSE THE GROOVES *G* WERE FILLED WITH PACKING AND ASPHALTUM TO MAKE THEM WATERTIGHT

gradually to the pressure of collapse by means of the screw plunger. It was measured by three tested gauges of the Bourdon type. The arrangement for closing the tube for collapse is shown in Fig. 7. This was the result of several trials, and proved very effective and easy to use. The steel end plugs were turned so that the tube could slide over them with a close fit, but without strain. The length of the tube thus supported on the inside at each end was about one-eighth inch for the small tubes, and as large as one half-inch for the three-inch tubes. The groove left around the tube was for the packing. The end of the tube was wrapped with "friction" tape—the rubberized tape used for insulation in electrical wiring—and the surrounding groove was packed with it. Melted asphaltum was then poured over the tape, and in some cases the whole end was coated by dipping it into the asphaltum. There was seldom any trouble with leakage even at the highest pressures. The end constraint was, by this arrangement, reduced practically to that of the support of the internal plug. To eliminate the end pressure on the tube, an internal bolt was used which was threaded at both ends and screwed down by means of tapped holes at the centers of the end plugs. As this bolt did not come through the end plugs, a serious source of leakage was avoided. The bolt or shaft was tapered so that it could be easily withdrawn from the collapsed tube. The upper plug was turned down and threaded so as to screw into a hole tapped into the large steel-disk cover of the nickel-steel receptacle. A small canal through this connected the interior of the tested tube with a manometer. The pressure on the inside was atmospheric except at the moment of collapse. The steel and brass tubes were carefully machined to the required thicknesses, so that results have been obtained for several values of $\frac{t}{d}$ for each length of tube.

Three gauges were used, the connections being such that any one or all could be used in each case. Two of the gauges were made by Shaffer and Budenberg of Magdeburg, Germany, the third by the Crosby Gage Company. The first gauge read to a maximum of 300 kilograms per square centimeter, each scale division representing 5 kilograms per square centimeter; the second, to a maximum of 1000 kilograms per square centimeter, each scale division representing 10 kilograms per square centimeter; and the third to 2000 pounds per square inch, each scale division representing 50 pounds per square inch. These gauges were tested by using a Crosby Fluid Pressure Scale. This is the standard rotating piston balance made by the Crosby Gage Company for testing gauges up to a pressure of 25,000 pounds per square inch. The readings of the gauges were corrected in accordance with these tests; the corrections were so small that they could have been neglected for these experiments.

6. *Results.*—The results of the experiments are given in Tables 1 to 7 inclusive (pages 33 to 36) and in Figs. 8 to 14 inclusive of this

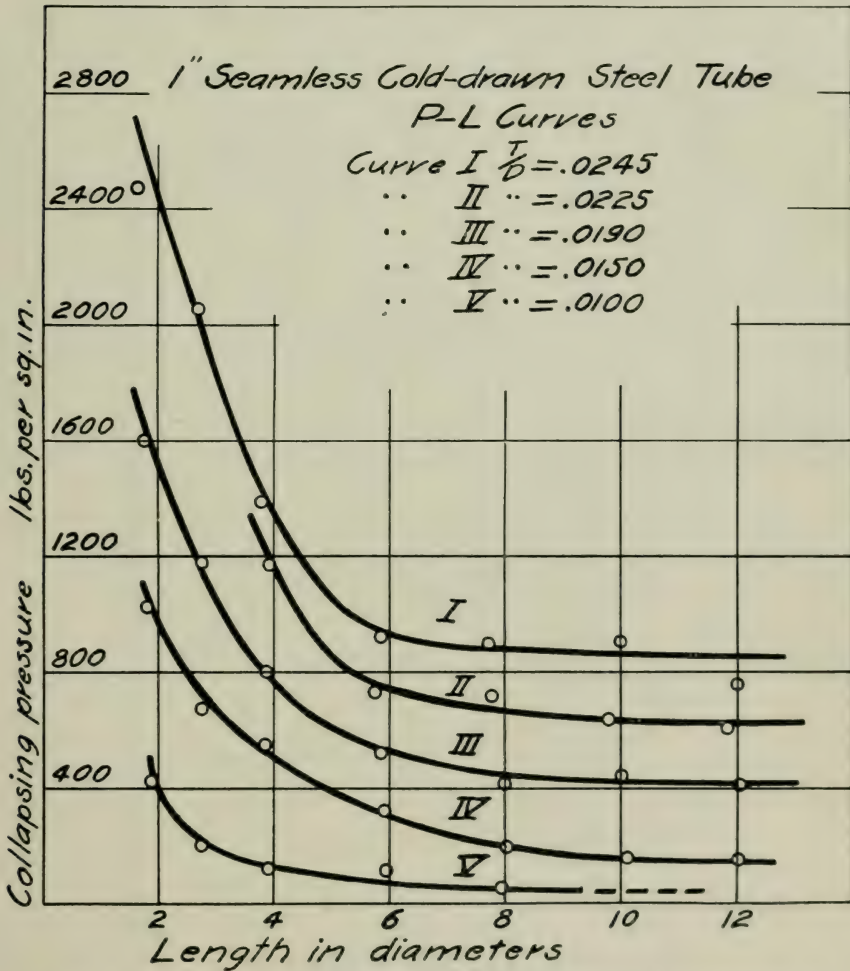

FIG. 8. PRESSURE-LENGTH CURVES FOR FIVE DIFFERENT THICKNESSES OF ONE-INCH SEAMLESS STEEL TUBES

bulletin. The number of lobes of the collapsed metal tubes is given in most cases. The number is recorded on account of its importance in the theory of stability and instability, which has been discussed by Southwell. In the collapse of glass tubes, we had the very striking

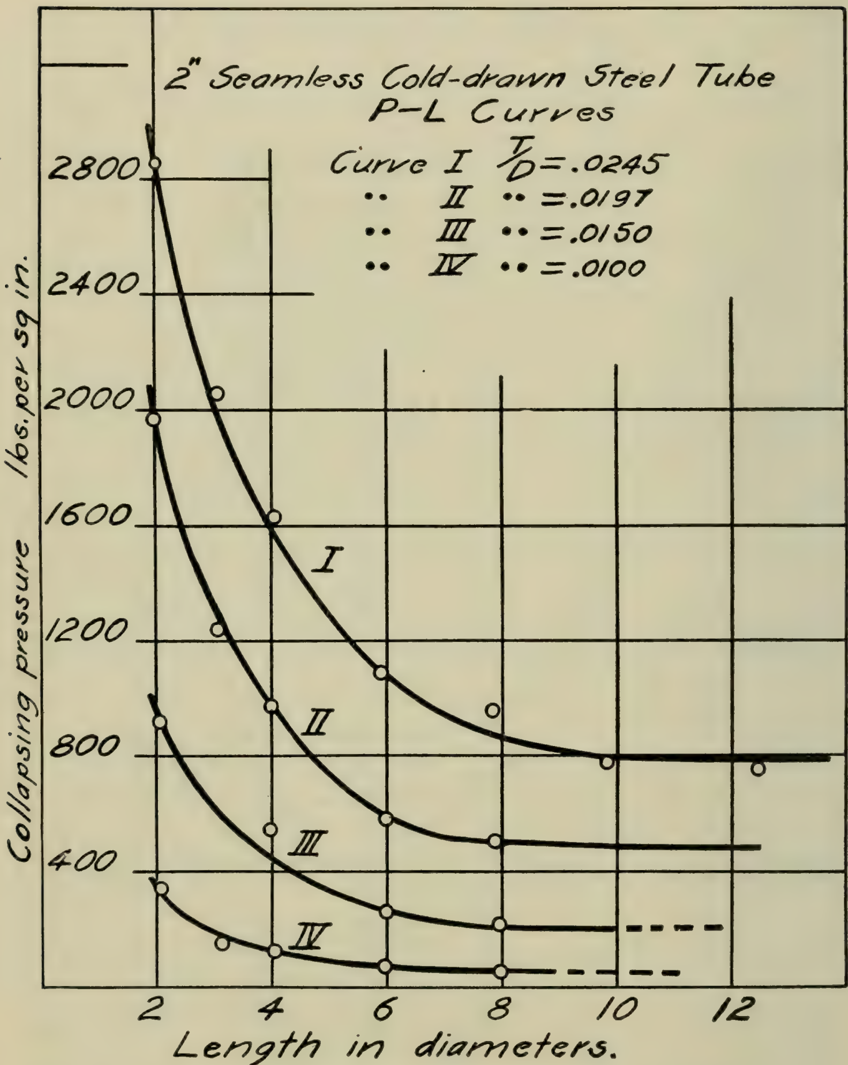

FIG. 9. PRESSURE-LENGTH CURVES FOR FOUR DIFFERENT THICKNESSES OF TWO-INCH SEAMLESS STEEL TUBES

occurrence of the glass being reduced to a fine powder. The data as given represent the results of the collapse of about one hundred and fifty tubes. In nearly all cases of the steel and the brass tubes, there were two tubes of the given dimensions; thus an average could be made for each point. This duplication also gave an immediate check on

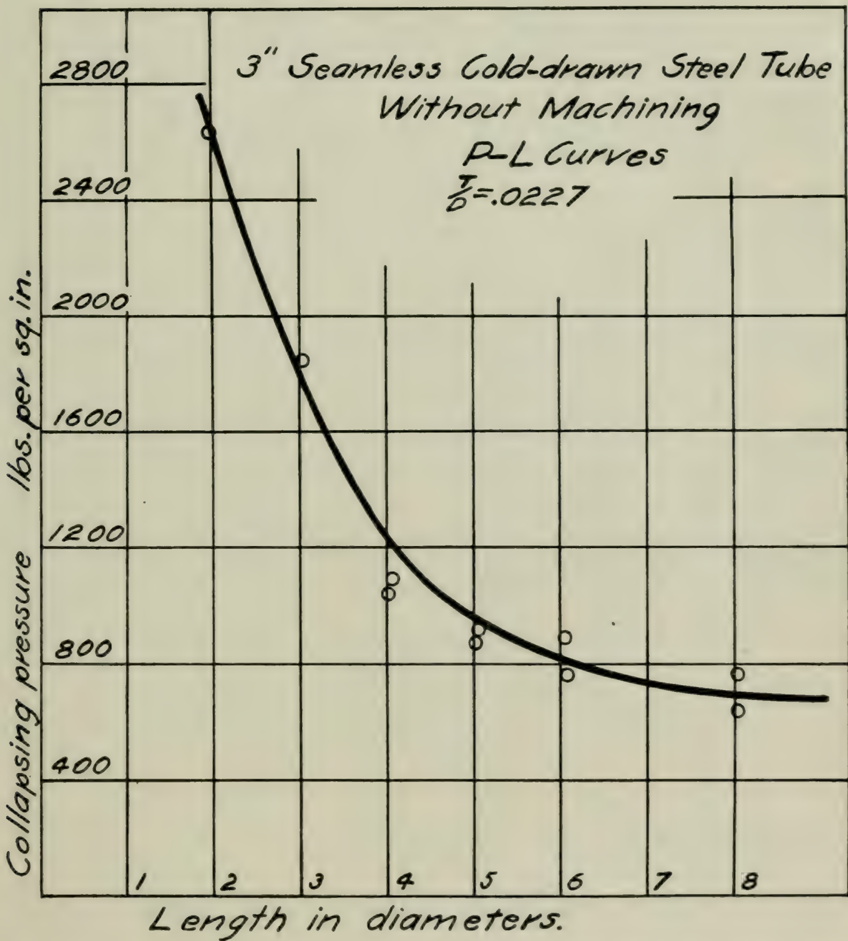

FIG. 10. PRESSURE-LENGTH CURVE FOR A THREE-INCH SEAMLESS STEEL TUBE WITH THICKNESS OF $\frac{t}{d}$ =.0227

freak collapses. Very few freak collapses occurred, however, and these could almost always be explained by the irregularities in dimensions or in material that appeared upon the inspection of the collapsed tube. While the machine work on these tubes was done with great care by the mechanicians of the department, Messrs. Hays and Buchanan, it was impossible to get the value of $\frac{t}{d}$ exactly the same each time. It was necessary therefore to make corrections in the

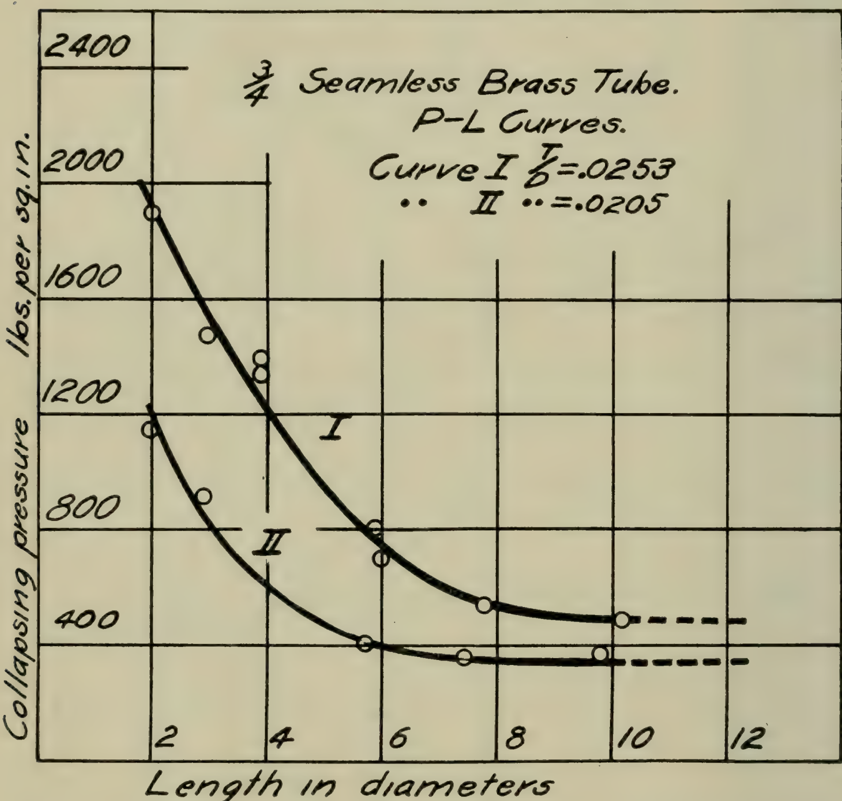

FIG. 11. PRESSURE-LENGTH CURVE FOR THREE-QUARTER-INCH BRASS TUBE WITH THICKNESS OF $\frac{t}{d}$ = .0205 AND .0253

observed collapsing pressures so as to have sets of results for a pressure-length curve with $\frac{t}{d}$ constant. These corrections were made by interpolation; it was assumed that the collapsing pressures varied as $(\frac{t}{d})^3$ for constant length. Since the total variations of $\frac{t}{d}$ in these cases were very small, there is little assumption in the interpolations and corrections.

Observations were also made of about fifty steel tubes of half-inch and three-quarter-inch diameters. The results were very irregular, due partly to unavoidable variations in thickness, but more to a lack of homogeneity of the material. Although the tubes were called "seamless," they showed a longitudinal seam upon being machined, an evidence of their non-uniformity. An attempt was made to anneal some

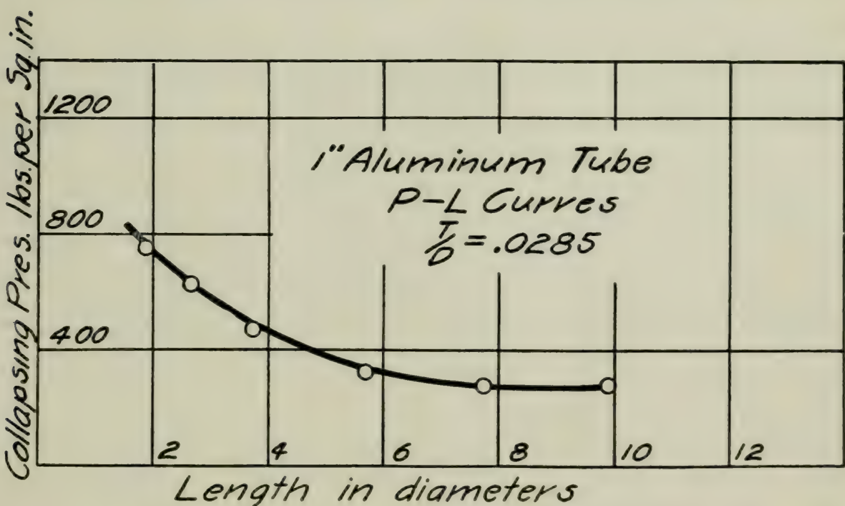

FIG. 12. PRESSURE-LENGTH CURVE FOR ONE-INCH ALUMINUM TUBE WITH THICKNESS OF $\frac{t}{d}$ = .0285

of the steel tubes to secure greater uniformity, but this introduced great irregularities of form. The tubes were packed inside an iron box and heated to about 1400 degrees F. (measured by a thermocouple) in the large gas furnace of the Mechanical Engineering Department and then allowed to cool slowly. The circular tubes came out quite elliptical, and annealing was therefore abandoned. The tubes used were purchased through dealers in the open market. The steel tubes were Shelby cold drawn steel tubes made by the National Tube Company. As these tubes are not always straight and are liable to vary in wall thickness, the dealers kindly helped us in selecting tubes of as perfect form as could be found in stock. Through the kindness of the laboratory of Applied Mechanics of this University we got the following elastic constants of the steel and brass:

For steel,	Young's modulus . .	29.94×10^6 pounds per sq. in.
	Yield point . . .	$5.96^6 \times 10^4$ pounds per sq. in.
	Ultimate strength . .	7.25×10^4 pounds per sq. in.
For brass,	Young's modulus . .	13.90×10^6 pounds per sq. in.
	Yield point . . .	7.45×10^4 pounds per sq. in.
	Ultimate strength . .	9.90×10^4 pounds per sq. in.

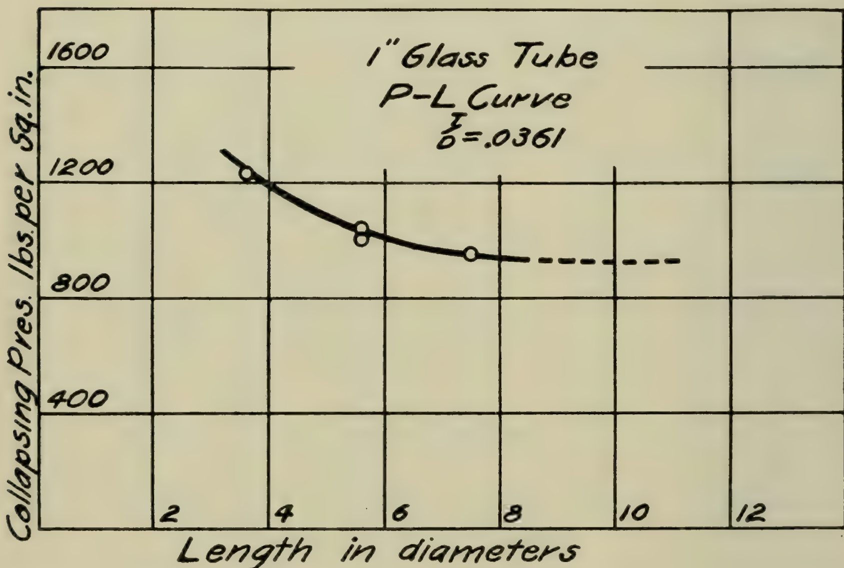

FIG. 13. PRESSURE-LENGTH CURVE FOR ONE-INCH GLASS TUBE WITH THICKNESS OF $\frac{t}{d} = .0361$

7. *Conclusions.*—A study of the curves leads to the following conclusions:

a. The pressure-length curves for different thicknesses and diameters are similar in shape. The series of curves for the one-inch and two-inch steel, the three-quarter-inch brass and the inch aluminum tubes show similar forms. The curves for glass and hard rubber differ from the others, the curve for the glass tubes bending more slowly, and that for the hard-rubber bending at a shorter length and more rapidly. The number of experiments on these last two materials is, however, too small to justify much generalization.

b. By taking, in the case of the metal tubes, the point for the length of six diameters, and drawing a hyperbola through this point using the equation $p' = p\frac{L}{l}$, there is in most cases a satisfactory agreement between the observed and the calculated curves for lengths of tubes less than six diameters. In Figs. 15, 16, and 17, the parts of

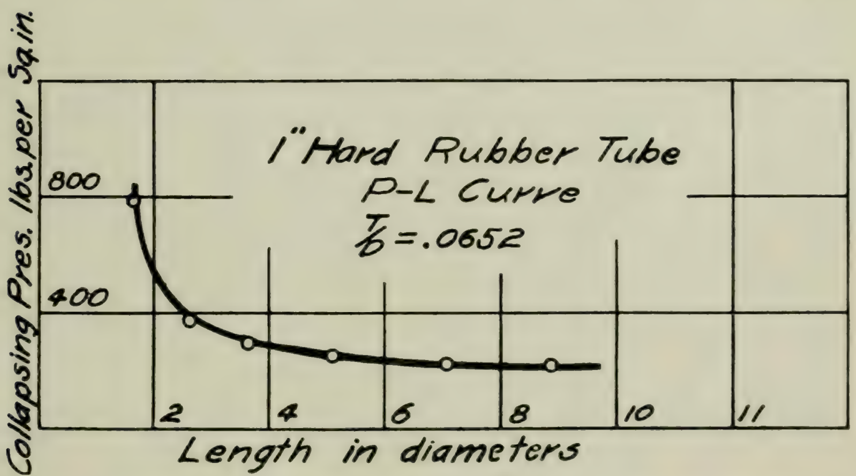

FIG. 14. PRESSURE-LENGTH CURVE FOR ONE-INCH HARD RUBBER TUBE WITH THICKNESS $\frac{t}{d}$ = .0652

the curves for lengths less than six diameters have been reproduced from Figs. 8, 9, and 10, and the calculated hyperbolas have been drawn on them. In a few cases the agreement of the two curves is very close; in one case the two curves are identical on the scale of the figure. In Figs. 18, 19, and 20, the observed curve and the calculated curves have been drawn and continued to lengths greater than six diameters. It is seen that the hyperbolas thus calculated show pressures less than those given by experiment. This length of six diameters, then, appears to be a "critical length." The experimental curve at this length bends rapidly towards the horizontal, particularly in the case of the thicker tubes. In the case of the thinnest steel tubes, both for those of one-inch and of two-inch diameters, that is, for tubes

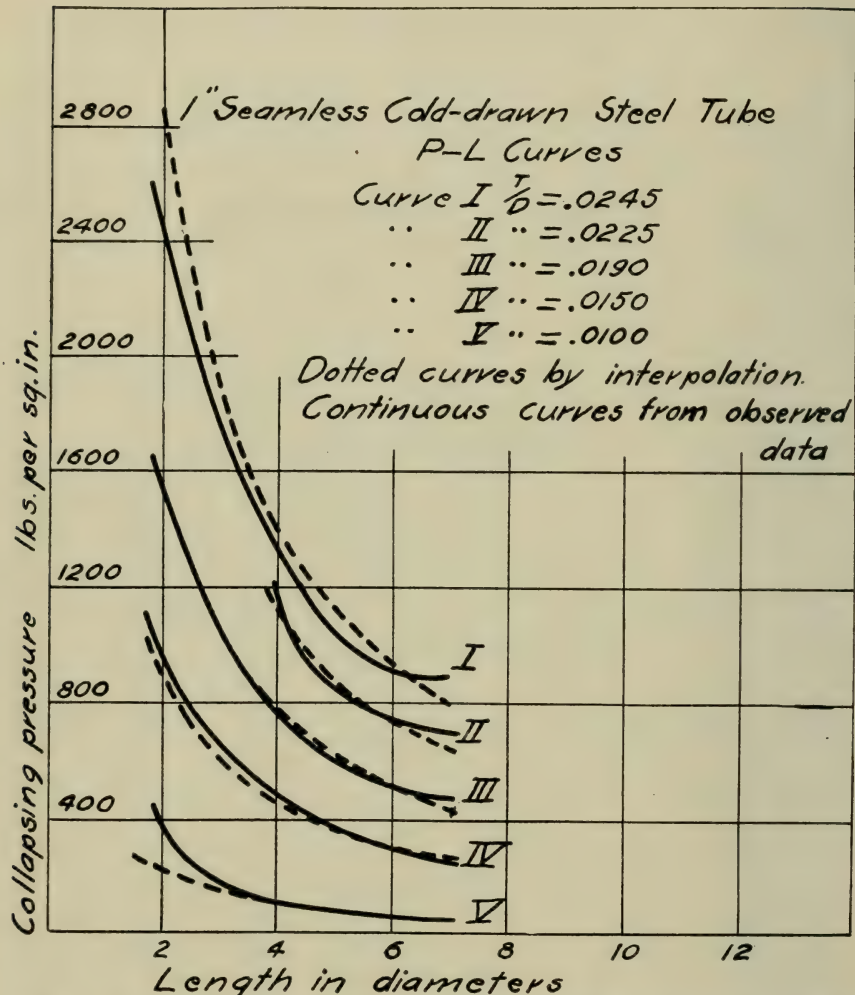

FIG. 15. PRESSURE-LENGTH CURVES OF ONE-INCH STEEL TUBES, PARTS OF THE CURVES FOR LENGTHS LESS THAN SIX DIAMETERS, COMPARED WITH HYPERBOLAS

in which the ratio of $\frac{t}{d}$ is about .001, the agreement with the hyperbola is not so exact; indeed, the maximum bend seems to occur at a much shorter length. While the curves for these very thin tubes show much uniformity, and the same characteristics are found in tubes of

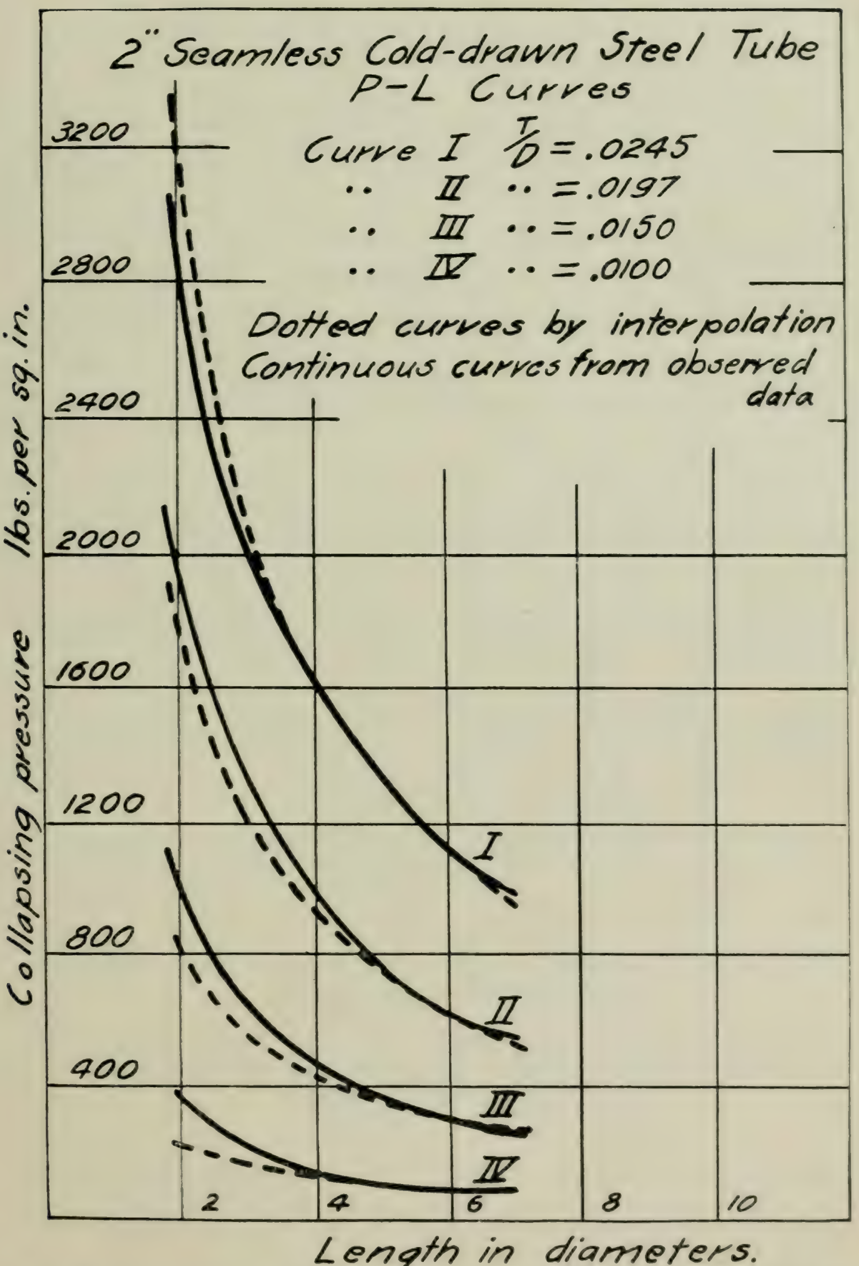

FIG. 16. PRESSURE-LENGTH CURVES OF TWO-INCH STEEL TUBES, PARTS OF THE CURVES FOR LENGTHS LESS THAN SIX DIAMETERS, COMPARED WITH HYPERBOLAS

different diameters, the percentage of error with such low pressures of collapse is necessarily greater.

c. The experimental curves of this investigation are not in agree-

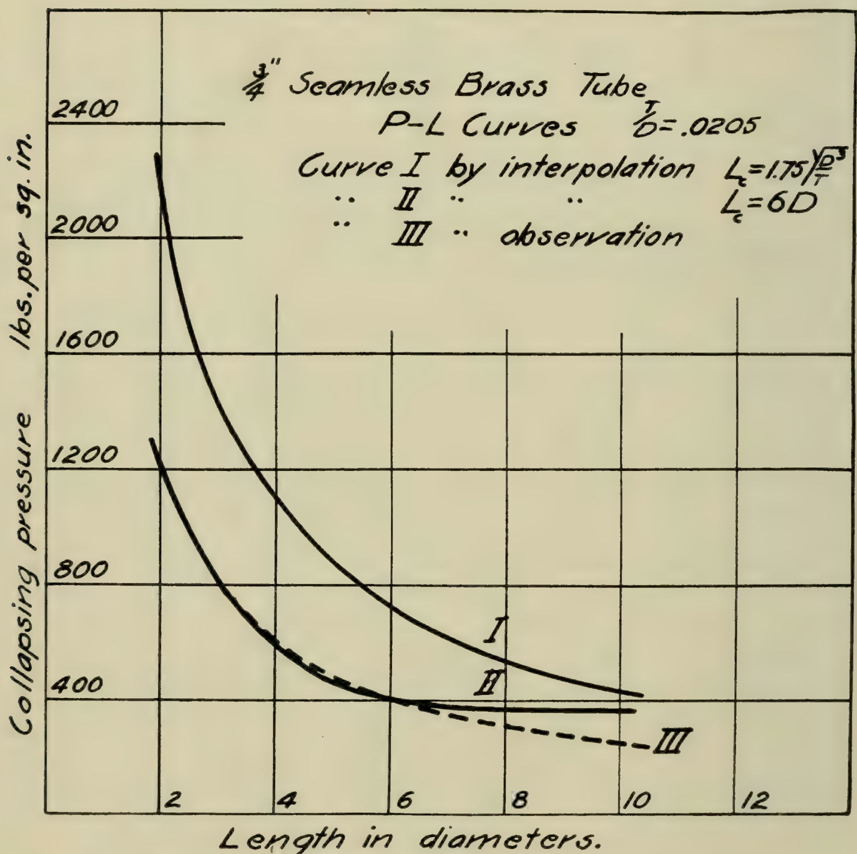

FIG. 17. PRESSURE-LENGTH CURVES OF THREE-QUARTER-INCH BRASS TUBES, PARTS OF THE CURVES FOR LENGTHS LESS THAN SIX DIAMETERS, COMPARED WITH HYPERBOLAS

ment with the Southwell formula $L = k\sqrt{\frac{d^3}{t}}$ which Cook has used. This is shown in typical cases in Figs. 18, 19, and 20, where the curves for $L=1.75\sqrt{\frac{d^3}{t}}$ are drawn for the one-inch and three-inch steel tubes

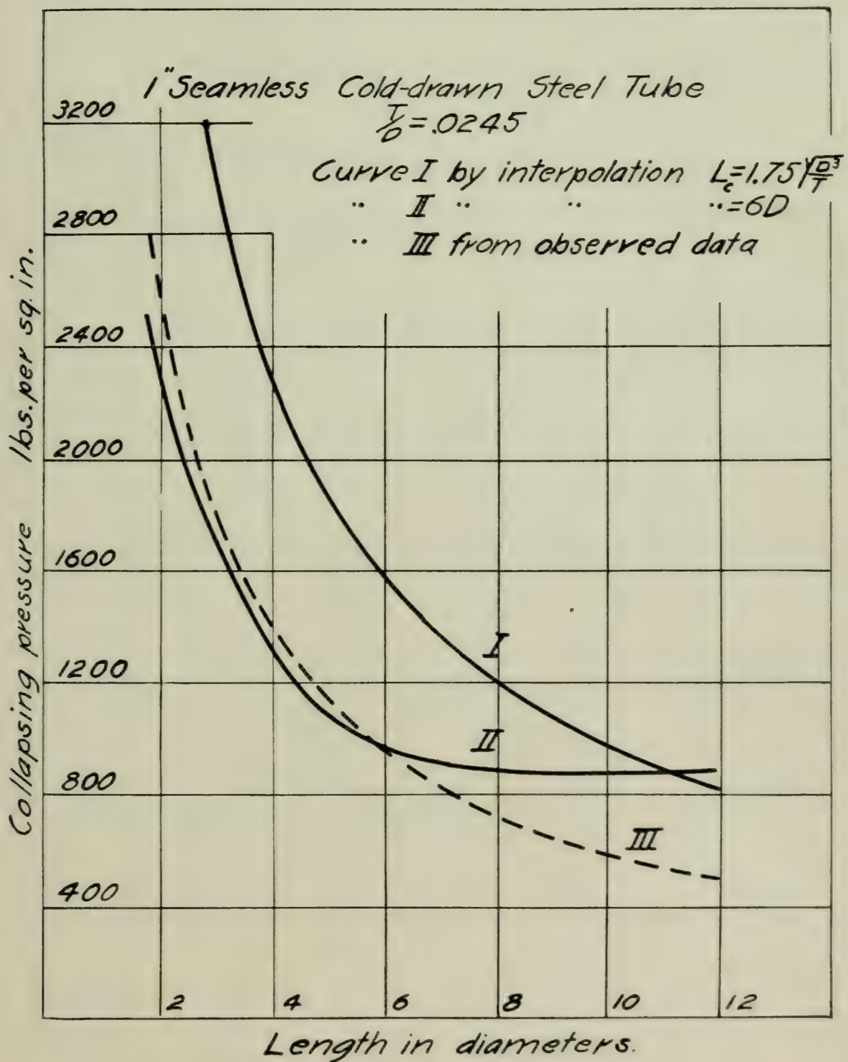

FIG. 18. PRESSURE-LENGTH CURVE OF ONE-INCH STEEL TUBE. EXPERIMENTAL CURVE COMPARED WITH TWO CALCULATED CURVES

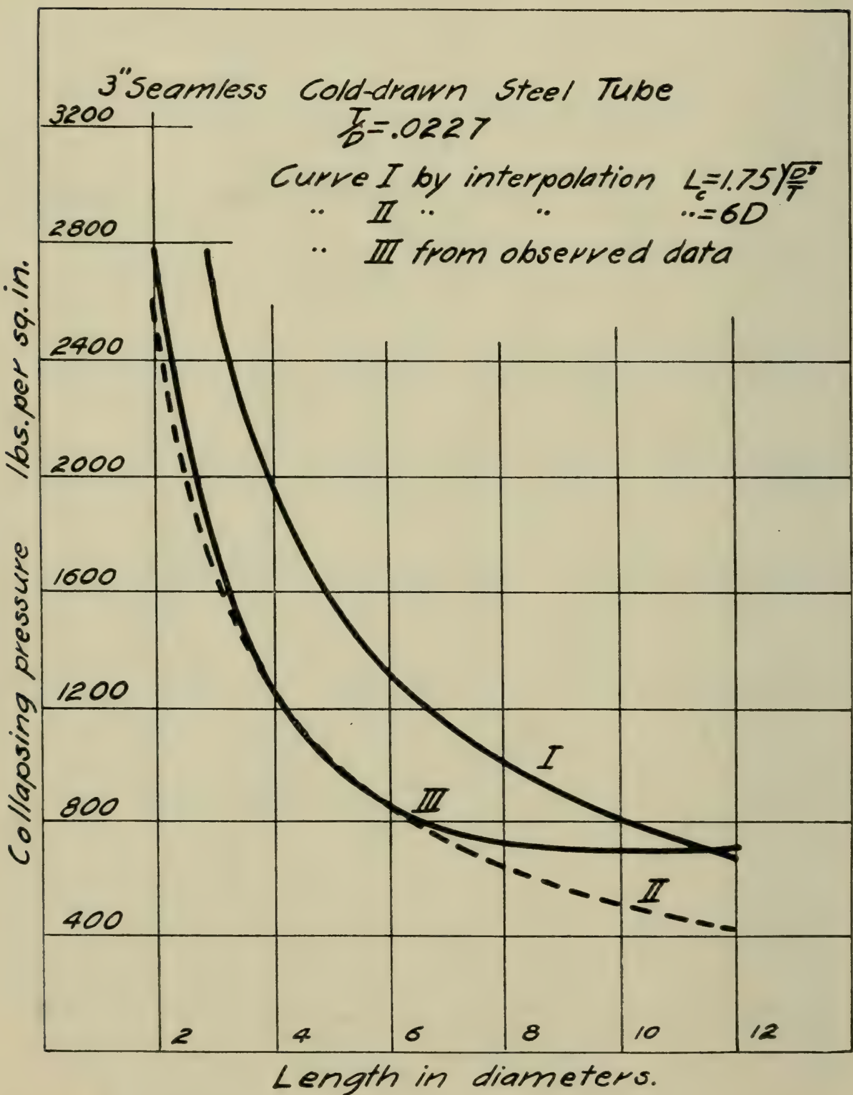

FIG. 19. PRESSURE-LENGTH CURVE OF THREE-INCH STEEL TUBE COMPARED WITH TWO CALCULATED CURVES

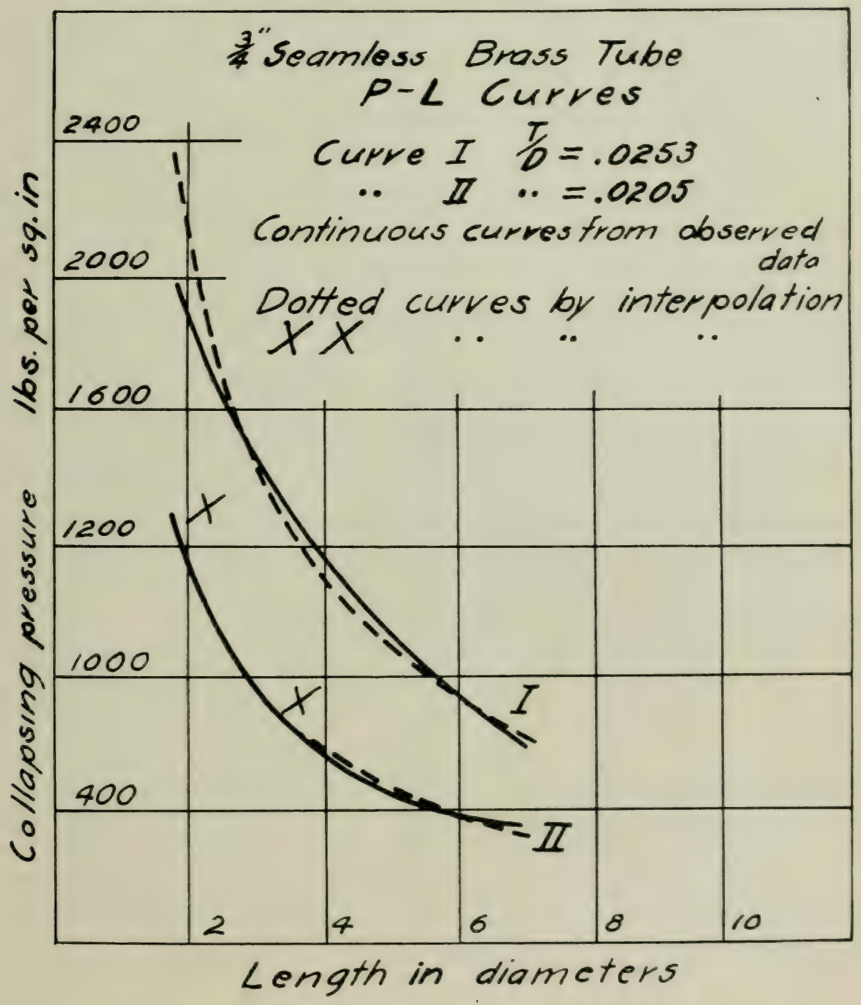

FIG. 20. PRESSURE-LENGTH CURVES OF THREE-QUARTER-INCH BRASS TUBES COMPARED WITH TWO CALCULATED CURVES

having a ratio $\frac{t}{d}$ of .0245 and .0227, and for the three-quarter-inch brass tube having a ratio $\frac{t}{d}$ of .0205. In the same figures the rectangular hyperbolas are drawn through the experimental point for a length of six diameters. If the formula $L = k\sqrt{\frac{d^3}{t}}$ is written in the form

$$L = \frac{kd}{\sqrt{\frac{t}{d}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

the critical length should increase directly as the diameter and inversely as the square root of $\frac{t}{d}$. It is evident that the length corresponding to the critical bend does vary as the diameter of the tube, but the curves do not show that the thinner tubes have longer critical lengths. Indeed our curves for the very thin one-inch and two-inch steel tubes rather indicate a shorter critical length than six diameters. To obtain reliable results on very thin tubes, facilities are required for testing tubes of large diameters, so that small variations in thickness and material may not affect the final results.

The experimental curves for these metal tubes for which the ratio $\frac{t}{d}$ is between .025 and .015 show that there is a ''critical'' bend at the length of about six diameters, the part of the curve for shorter lengths being approximately a rectangular hyperbola. Beyond this critical bend the curve approaches more or less rapidly a straight line parallel to the axis of lengths. These results, in connection with previous results on the collapsing pressures of long tubes, make it possible to calculate from the data with reasonable approximation the collapsing pressures of short tubes of certain thicknesses.

TABLE 1

ONE-INCH STEEL TUBES

INSIDE DIAMETER .942 INCHES

Length in Inches	Length in Diams.	$\frac{t}{d}$	Pressure of Collapse Lb. per Sq. In.	Number of Lobes
1.72	1.8	.0245	2490	4
1.74	1.8	.0190	1600	3 or 4
1.785	1.86	.0150	1030	3 or 4
1.78	1.87	.01	420	4
2.63	2.73	.0245	2060	3
2.63	2.74	.0190	1180	3
2.63	2.74	.0150	670	3
2.63	2.75	.010	200	3
3.69	3.82	.0245	1390	2 or 3
3.69	3.84	.0190	800	3
3.69	3.85	.0150	550	3
3.69	3.9	.010	120	3
3.72	3.9	.0225	1170	3
5.63	5.85	.0245	920	2
5.63	5.73	.0225	730	2
5.63	5.85	.0190	520	2
5.63	5.9	.0150	320	2
5.63	5.9	.010	115	2
5.5	5.6	.0227	675	3
7.5	7.8	.0245	900	2
7.59	7.7	.0225	710	2
7.56	8.	.010	50	2
7.44	7.8	.015	210	2
7.62	8.	.015	190	2
7.62	8.	.019	420	2
9.6	10.	.0245	900	2
9.56	9.75	.0225	625	2
9.6	10.	.019	440	2
9.56	10.	.015	150	2
11.6	12.	.0245	750	2
11.63	11.85	.0225	600	2
11.63	12.	.019	375	2
11.6	12.1	.015	145	2

TABLE 2

TWO-INCH STEEL TUBES

INSIDE DIAMETER 1.873 INCHES

Length in Inches	Length in Diams.	$\frac{t}{d}$	Pressure of Collapse Lb. per Sq. In.	Number of Lobes
3.91	2.	.0245	2870	3
3.88	2.	.0197	1770	3
4.	2.1	.015	920	3
3.88	2.1	.010	350	2
5.94	3.1	.0245	2100	3
5.94	3.1	.0197	1250	3
5.94	3.1	.015	580	3
5.94	3.1	.010	150	2
7.8	4.1	.0245	1630	2
7.63	4.	.0197	980	2
7.63	4.	.015	540	2
7.66	4.	.010	135	2
11.34	5.9	.0245	1080	2
11.34	6.	.0197	590	2
11.34	6.	.015	260	2
11.34	6.	.010	70	2
15.13	7.9	.0245	960	2
15.13	7.9	.0197	510	2
15.13	8.	.0150	220	2
15.13	8.	.0099	55	2
18.94	9.9	.0245	794	2
23.0	12.5	.0245	780	2
27.8	14.5	.0245	855	2

TABLE 3

THREE-INCH STEEL TUBES

INSIDE DIAMETER 2.87 INCHES

Length in Inches	Length in Diams.	$\frac{t}{d}$	Pressure of Collapse Lb. per Sq. In.	Number of Lobes
5.88	2.	.0227	2640	4
9.12	3.1	.0227	1960	..
12.14	4.1	.0227	1390	2
12.17	4.1	.0227	1040	2
12.19	4.1	.0227	1010	2
15.06	5.1	.0227	873	2
15.06	5.1	.0227	910	2
18.13	6.2	.0227	756	2
18.28	6.2	.0227	895	2
24.13	8.2	.0227	684	2
24.25	8.2	.0227	770	2

TABLE 4

THREE-QUARTER-INCH BRASS TUBES
INSIDE DIAMETER .751 INCHES

Length in Inches	Length in Diams.	$\frac{t}{d}$	Pressure of Collapse Lb. per Sq. In.	Number of Lobes
1.56	2.	.0138	310	3
1.56	2.	.0138	400	3
1.56	2.	.0205	1150	3
1.56	2.	.0253	1900	3
2.28	3.	.0138	345	..
2.28	3.	.0138	300	..
2.28	2.9	.0205	920	..
2.25	2.9	.0253	1470	..
2.25	2.9	.0253	1310	..
3.	3.9	.0253	1400	..
3.	3.9	.0253	1360	..
3.	3.9	.0276	1420	..
4.	5.7	.0205	409	2
4.	5.7	.0205	429	2
4.56	5.9	.0253	810	2
4.63	6.	.0253	700	2
5.69	7.4	.0205	348	2
6.	7.8	.0205	550	2
7.5	9.8	.0205	362	2
7.5	9.8	.0205	388	2

TABLE 5

ONE-INCH ALUMINUM TUBES
INSIDE DIAMETER .944 INCHES

Length in Inches	Length in Diams.	$\frac{t}{d}$	Pressure of Collapse Lb. per Sq. In.	Number of Lobes
1.18	1.9	.028	750	4
2.59	2.7	.0286	630	3
3.63	3.7	.0285	470	3
3.63	3.7	.029	475	3
5.53	5.7	.0288	330	2
7.53	7.8	.029	280	2
9.59	9.9	.0295	280	1

TABLE 6

ONE-INCH GLASS TUBES

INTERNAL DIAMETER .96 INCHES

Length in Inches	Length in Diams.	$\frac{t}{d}$	Pressure of Collapse Lb. per Sq. In.	Condition of Collapsed Tube
3.59	3.7	.036	1230	practically pulverized
5.63	5.8	.036	1040	" "
5.63	5.8	.036	1000	" "
7.47	7.7	.036	950	" "

TABLE 7

ONE-INCH HARD RUBBER TUBES

Length in Inches	Length in Diams.	$\frac{t}{d}$	Pressure of Collapse Lb. per Sq. In.	Condition of Collapsed Tube
1.69	1.6	.0643	790	One side caved in and broken in very fine parts.
2.63	2.5	.0654	370	In irregular parts
3.63	3.4	.0650	290	"
5.56	5.2	.0654	250	"
7.56	7.1	.0652	220	"
9.55	8.9	.0645	220	"

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